

RATIONAL EXPRESSIONS AND EQUATIONS

MULTIPLICATION AND DIVISION OF FRACTIONS

$$\frac{A}{B} \cdot \frac{C}{D} = \frac{A \cdot C}{B \cdot D}$$

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C} = \frac{A \cdot D}{B \cdot C}$$

Example: Multiply the following fractions

$$\begin{aligned} & \frac{4}{7} \cdot \frac{5}{6} \\ &= \frac{4 \cdot 5}{7 \cdot 6} \\ &= \frac{20}{42} \\ &= \frac{20 \div 2}{42 \div 2} \quad \text{Reduce} \\ &= \frac{10}{21} \end{aligned}$$

Example: Divide the following

$$\begin{aligned} & \frac{14}{9} \div \frac{7}{6} \\ &= \frac{14}{9} \cdot \frac{6}{7} \\ &= \frac{14 \cdot 6}{9 \cdot 7} \\ &= \frac{2 \cdot 2}{3 \cdot 1} \\ &= \frac{4}{3} \end{aligned}$$

In general, the multiplication or division of rational expressions follows the same logic as simple fractions. The only requirement is the factorization of both numerator and denominator, and simplification of identical terms between them.

Example: Multiply the following fractions

$$\begin{aligned} & \frac{x^2 - x - 6}{3x} \cdot \frac{12x^3}{x + 2} \\ &= \frac{(x^2 - x - 6)(12x^3)}{3x(x + 2)} \\ &= \frac{(x - 3)(x + 2)(3x)(4x^2)}{3x(x + 2)} \\ &= 4x^2(x - 3) \end{aligned}$$

Such simplification is only possible when $3x \neq 0$ and $x + 2 \neq 0$.

Example: Divide the following

$$\begin{aligned} & \frac{x^2 - 1}{x + 5} \div \frac{x^2 + 2x + 1}{2x + 10} \\ &= \frac{x^2 - 1}{x + 5} \cdot \frac{2x + 10}{x^2 + 2x + 1} \\ &= \frac{(x - 1)(x + 1)(2)(x + 5)}{(x + 5)(x + 1)(x + 1)} \\ &= \frac{2(x - 1)}{x + 1} \end{aligned}$$

Such simplification is only possible when $x + 5 \neq 0$ and $x + 1 \neq 0$.

EXERCISE: Perform each indicated operation, and if possible, simplify.

(1) $\frac{3x}{x+y} \cdot \frac{2x+2y}{x^2}$

(2) $\frac{a^2+2a+1}{a} \div \frac{a^2-1}{a^2}$

(3) $\frac{x^2+4x+3}{x^2+x-2} \cdot \frac{x^2+3x+2}{x^2+2x-3}$

ADDITION AND SUBTRACTION OF FRACTIONS – LIKE FRACTIONS

$\frac{3}{5}$ and $\frac{7}{5}$ are like fractions; $\frac{3}{2x}$ and $\frac{-5}{2x}$ are also like fractions; However, $\frac{3}{5}$ and $\frac{3}{7}$ are NOT like fractions; To add or subtract like fractions, just add or subtract the numerators and keep the denominator the same.

$$\frac{A}{B} + \frac{C}{B} = \frac{A + C}{B}$$

$$\frac{A}{B} - \frac{C}{B} = \frac{A - C}{B}$$

Example: Add the following fractions

$$\begin{aligned}\frac{4}{11} + \frac{5}{11} \\&= \frac{4 + 5}{11} \\&= \frac{9}{11}\end{aligned}$$

Example: Subtract the following fractions

$$\begin{aligned}\frac{4}{11} - \frac{5}{11} \\&= \frac{4 - 5}{11} \\&= \frac{-1}{11}\end{aligned}$$

Example: Add the following fractions

$$\begin{aligned}\frac{x - 6}{x^2 - x - 6} + \frac{x^2 - 6}{x^2 - x - 6} \\&= \frac{(x - 6) + (x^2 - 6)}{x^2 - x - 6} \\&= \frac{x^2 - x - 6}{x^2 + x - 12} \\&= \frac{x^2 - x - 6}{(x + 4)(x - 3)} \\&= \frac{(x - 3)(x + 2)}{(x - 3)(x + 2)} \\&= \frac{x + 4}{x + 2} \quad \text{for } x \neq 3\end{aligned}$$

ADDITION AND SUBTRACTION OF FRACTIONS – UNLIKE FRACTIONS

To ADD or SUBTRACT unlike fractions you need to reduce them to the same denominator using the LCD method.

Example: Add the following fractions

$$\begin{aligned} & \frac{4}{15} + \frac{2}{5} \\ &= \frac{4}{15} + \frac{2 \cdot 3}{5 \cdot 3} && \text{Rewrite the second fraction to the same denominator} \\ &= \frac{4}{15} + \frac{6}{15} \\ &= \frac{4+6}{15} \\ &= \frac{10}{15} \\ &= \frac{10 \div 5}{15 \div 5} && \text{Reduce} \\ &= \frac{2}{3} \end{aligned}$$

Example: Add the following fractions

$$\begin{aligned} & \frac{x-3}{x^2-1} + \frac{4x^2}{x^2+4x+3} \\ &= \frac{(x-3)}{(x-1)(x+1)} \cdot \frac{(x+3)}{(x+3)} + \frac{4x^2}{(x+1)(x+3)} \cdot \frac{(x-1)}{(x-1)} \\ &= \frac{x^2-9+4x^3-4x^2}{(x-1)(x+1)(x+3)} \\ &= \frac{4x^3-3x^2-9}{(x-1)(x+1)(x+3)} \end{aligned}$$

Example: Subtract the following fractions

$$\begin{aligned} & \frac{x}{x+2} - \frac{2x-3}{3x-4} \\ &= \frac{x}{x+2} \cdot \frac{3x-4}{3x-4} - \frac{2x-3}{3x-4} \cdot \frac{x+2}{x+2} && \text{We multiply the first fraction by } 3x-4 \text{ (both numerator and denominator), and the second by } x+2. \\ &= \frac{x(3x-4) - (2x-3)(x+2)}{(3x-4)(x+2)} \\ &= \frac{3x^2-4x-2x^2-4x+3x+6}{(3x-4)(x+2)} \\ &= \frac{x^2-5x+6}{(3x-4)(x+2)} && \text{Distribute the numerator} \end{aligned}$$

Example: Divide the following fractions

$$\begin{aligned} & \frac{\frac{2}{x+1}}{\frac{1}{x+2} + \frac{1}{x}} \\ &= \frac{\frac{2}{x+1} \cdot \left(\frac{(x+1)(x+2)x}{1} \right)}{\frac{1}{x+2} \cdot \left(\frac{(x+1)(x+2)x}{1} \right) + \frac{1}{x} \cdot \left(\frac{(x+1)(x+2)x}{1} \right)} && \text{Find the LCD of all simple fractions involved, that is here, } \frac{2}{x+1}, \frac{1}{x+2} \text{ and } \frac{1}{x}. \text{ That LCD is } (x+1)(x+2)x. \text{ So we multiply each simple fraction by } \frac{(x+1)(x+2)x}{1} \\ &= \frac{2(x+2)x}{(x+1)x + (x+1)(x+2)} \\ &= \frac{2x^2+4x}{2x^2+4x+2} \\ &= \frac{2(x^2+2x)}{2(x^2+2x+1)} \\ &= \frac{x(x+2)}{(x+1)^2} \end{aligned}$$

EXAMPLE:

$$(4) \frac{2x}{x-5} + \frac{3}{x+4}$$

$$(5) \frac{x^3+2x^2+x}{x^2-4} \cdot \frac{x^2-x-2}{x^4+x^3}$$

$$(6) \frac{-1}{x^2+7x+10} - \frac{3}{x^2+8x+15}$$

$$(7) \frac{z^2+2z+1}{8z} \div \frac{z^2-z-2}{4z^2-4}$$

$$(8) \frac{x-\frac{3}{x-2}}{x-\frac{12}{x+1}}$$

$$(9) \frac{\frac{x-3}{3-x}}{\frac{1}{x+1} + \frac{1}{3}}$$

$$(10) \frac{\frac{x}{2y^2} + \frac{y}{2x^3}}{\frac{1}{6xy} + \frac{2}{x^2y}}$$

$$(11) \text{ Solve } \frac{3}{x+7} = \frac{1}{x-8}$$

$$(12) \text{ Solve } \frac{1}{x-3} - \frac{x-4}{x^2-9} = 1$$

(13) Lindsay can build a fence in 6 hr. Laura can do the same job in 4hr. How long will it take them, working together, to build the fence?

(14) Ani bicycles 8 mi and Lia bicycles 12 mi to meet at a park for lunch. Because Ani's trip is mostly uphill, she rides 5 mph slower than Lia. Ani and Lia leave their homes at the same time and arrived at the park at the same time. Find the speed of each bicyclist.

Answers

- 1) $\frac{6}{x}$
- 2) $\frac{a(a+1)}{a-1}$
- 3) $\frac{(x+1)^2}{(x-1)^2}$
- 4) $\frac{2x^2+11x-15}{(x-5)(x+4)}$
- 5) $\frac{(x+1)^2}{x^2(x+2)}$
- 6) $\frac{-(4x+9)}{(x+2)(x+3)(x+5)}$
- 7) $\frac{(z+1)^2(z-1)}{2z(z-2)}$
- 8) $\frac{(x+1)^2}{(x-2)(x+4)}$
- 9) $x - 3$
- 10) $\frac{3(x^4+y^3)}{xy(x+12)}$
- 11) $x = 31/2$
- 12) $x = -4 \text{ and } x = 4$
- 13) 2.4 hours
- 14) Ani's speed is 10 mph and Lia's speed is 15 mph.