SOLVING LINEAR EQUATIONS IN ONE VARIABLE

LINEAR EQUATIONS IN ONE VARIABLE

Examples of linear equations in one variable.

$$x + 2 = 7 \qquad\qquad \frac{2a}{9} + a = \frac{22}{15}$$

Any numerical value for a variable that makes an equation a true statement is called a **solution of the equation**. To solve an equation means to find all of its solution.

Example: $\boldsymbol{x + 2 = 7}$

a) Is $\boldsymbol{5}$ a solution to the above equation?

$5 + 2 = 7$
$7 = 7 \quad True\ \ statement$
Therefore, 5 is a solution.

b) Is $\boldsymbol{-1}$ a solution to the above equation?

$-1 + 2 = 7$
$1 = 7 \quad False\ statement$
Therefore, -1 is a not a solution.

When solving a linear equation in one variable, one of the outcomes will occur:

1. Obtain exactly one solution. Solution set: $\{x_1\}$
2. If the solution process results in a true statement for any value of the variable, then all real numbers are solutions to the equation. Solution set: $(-\infty, \infty)$
3. If the solution process results in a false statement, then there are no solutions to the equation. Solution set: $\{\ \}$

I. THE ADDITION PRINCIPLE

If $a = b$ then $a + c = b + c$, for any real-valued a, b, c.

Example: $x + 2 = 7$

$x + 2 - 2 = 7 - 2$

$x = 5$

Solution set: $\{5\}$

Example: $y - \frac{1}{4} = \frac{-1}{5}$

$y - \frac{1}{4} + \frac{1}{4} = \frac{-1}{5} + \frac{1}{4}$

$y = \frac{-4}{20} + \frac{5}{20}$

$y = \frac{1}{20}$

Solution set: $\left\{\frac{1}{20}\right\}$

EXERCISE:

(1) $-4+x=-21$ (2) $x+0.028=1$ (3) $\frac{3}{5}+x=-\frac{1}{40}$ (4) $-8.65=z+4.12$

(5) $y+8=14$ (6) $-15=y-6$ (7) $x-\frac{5}{12}=1$ (8) $-3.4+x=-6.08$

II. THE MULTIPLICATION PRINCIPLE

If $a=b$ then $a\cdot c=b\cdot c$, for any real-valued a,b,c where $c\neq 0$.

Example: $4x=32$

$$4x\cdot\frac{1}{4}=32\cdot\frac{1}{4}$$
$$\frac{4x}{4}=\frac{32}{4}$$
$$x=8$$

Solution set: $\{8\}$

Example: $\frac{-4y}{5}=14$

$$\frac{-4y}{5}\cdot\frac{5}{-4}=14\cdot\frac{5}{-4}$$
$$y=\frac{-35}{2}$$

Solution set: $\left\{\frac{-35}{2}\right\}$

EXERCISE:

(9) $\frac{-x}{5}=12$ (10) $0.02y=1$ (11) $\frac{2}{7}\cdot x=-\frac{1}{14}$ (12) $-56=-8z$

(13) $\frac{1}{4}x=-4$ (14) $\frac{1}{6}=-x$ (15) $-\frac{5}{12}x=1$ (16) $0.002x=0.8$

III. USING THE PRINCIPLES TOGETHER

1. If the equation contains parenthesis, first use the distributive property.
2. Combine like terms on each side of the equation.
3. Use the addition principle to isolate all variable terms on one side and non-variable (constant) terms on the other side.
4. Combine like terms on each side of the equation.
5. Use the multiplication principle to solve for the variable.
6. State the solution set.

Example: $1.2x + 54.2 = 2.3 - 4.8x$

$1.2x + 54.2 + 4.8x = 2.3 - 4.8x + 4.8x$

$6x + 54.2 - 54.2 = 2.3 - 54.2$

$6x = -51.9$

$\frac{6x}{6} = \frac{-51.9}{6}$

$x = -8.65 \quad \{-8.65\}$

Solution set: $\{-8.65\}$

Example: $\frac{1}{4}(5x - 1) = x + 7$

$\frac{5x}{4} - \frac{1}{4} = x + 7$

$\frac{5x}{4} - \frac{1}{4} - x = x + 7 - x$

$\frac{1}{4}x - \frac{1}{4} = 7$

$\frac{1}{4}x - \frac{1}{4} + \frac{1}{4} = 7 + \frac{1}{4}$

$\frac{1}{4}x \cdot 4 = \frac{29}{4} \cdot 4$

$x = 29$

Solution set: $\{29\}$

Example: $4(x - 3) + 20 = 6x - 2(x - 4)$

$4x - 12 + 20 = 6x - 2x + 8$

$4x + 8 = 4x + 8$

$4x + 8 - 4x = 4x + 8 - 4x$

$8 = 8$

True Statement for any value of x.

All real numbers are solutions.

Solution set: $(-\infty, \infty)$

Example: $5y - (y - 3) = 5(2y - 1) + 6$

$5y - y + 3 = 10y - 5 + 6$

$4y + 3 = 10y + 1$

$4y + 3 - 10y = 10y + 1 - 10y$

$-6y + 3 = 1$

$-6y + 3 - 3 = 1 - 3$

$-6y = -2$

$\frac{-6y}{-6} = \frac{-2}{-6}$

$y = \frac{1}{3}$

Solution set: $\left\{\frac{1}{3}\right\}$

EXERCISE:

(17) $y - 6y + 1 = 2y - 5 - 4y$

(18) $4x - 0.25x + 11.4 = 3x - 3.6$

(19) $8 - 5(3m + 8) = 7 - 5(m - 6)$

(20) $20 = 15 - \frac{x}{5}$

(21) $4 - (x - 12) = 0.8(7x + 4) + 2$

(22) $\frac{2}{5}\left(\frac{1}{4} - 3x\right) - \frac{1}{4} = \frac{1}{5}$

(23) $2x - 21 = 5x - 3(x + 7)$

(24) $5(x - 7) = 3(x - 2) + 2x$

(25) $\frac{-1}{4}x + x = \frac{-1}{6}x - \frac{11}{15}$ (26) $7.1t + 5 - 2.61t = 0.4t + 5.18 + 8$

IV. APPLICATIONS TO PROBLEM SOLVING

Example: The perimeter of a rectangle is 184 cm. The length of the rectangle is 4 cm longer than its width. Find the dimensions of the rectangle.

Width = w Length = $w + 4$

$w + (w + 4) + w + (w + 4) = 184$

$4w + 8 = 184$

$4w + 8 - 8 = 184 - 8$

$4w = 176$

$\frac{4w}{4} = \frac{176}{4}$

$w = 44$

Width = 44 cm Length = $44 + 4 = 48$ cm

Example: A bag contains pink and yellow marbles. The number of yellow marbles is 7 less than four times the number of pink marbles. If there are 38 marbles in the bag, how many of them are yellow?

Number of pink marbles = x Number of yellow marbles = $4x - 7$

$x + (4x - 7) = 38$

$5x - 7 + 7 = 38 + 7$

$5x = 45$

$\frac{5x}{5} = \frac{45}{5}$

$x = 9$

Number of yellow marbles = $4(9) - 7 = 36 - 7 = 29$

EXERICSE:

(27) The second angle of a triangle is two times as large as the first. The third angle is $48°$ less than the sum of the other two angles. Find the measure of each angle.

(28) Two angles are complementary. The measure of one angle is $2\frac{3}{4}$ times the measure of the other angle. Find the measure of each angle.

(29) An investment increased by 17% to \$468. What was the original investment?

(30) A phone plan consists of a monthly charge of \$ 24.70 and 15 *cents* for each text message sent or received. If the total bill for one month was \$40.00, how many texts were sent or received?

(31) Four consecutive even integers add up to 796. Find the integers.

Answers

1.) $\{-17\}$

2.) $\{0.972\}$
3.) $\left\{\frac{-5}{8}\right\}$
4.) $\{-12.77\}$
5.) $\{6\}$
6.) $\{-9\}$
7.) $\left\{\frac{17}{12}\right\}$
8.) $\{-2.68\}$
9.) $\{-60\}$
10.) $\{50\}$
11.) $\left\{\frac{-1}{4}\right\}$
12.) $\{7\}$
13.) $\{-16\}$
14.) $\left\{-\frac{1}{6}\right\}$
15.) $\left\{-\frac{12}{5}\right\}$
16.) $\{400\}$
17.) $\{2\}$
18.) $\{-20\}$
19.) $\{-6.9\}$
20.) $\{-25\}$
21.) $\left\{\frac{18}{11}\right\}$
22.) $\left\{-\frac{7}{24}\right\}$
23.) $(-\infty, \infty)$
24.) $\{\ \}$
25.) $\left\{\frac{-4}{5}\right\}$
26.) $\{2\}$
27.) $\{38^0, 76^0, 66^0\}$
28.) $\{24^0, 66^0\}$
29.) $\{\$400\}$
30.) $\{102\}$
31.) $\{196,198,200,202\}$